EJERCICIO RESUELTO

Calcula las siguientes integrales:

a) $\displaystyle\int \frac{x^4 - 3x^2 + 2x - 2}{x - 1}\,dx$ *b)* $\displaystyle\int \frac{\sqrt[3]{x} + \sqrt{2x^2}}{\sqrt[4]{x}}\,dx$ *c)* $\displaystyle\int \frac{2sen\,x + 3e^x}{5}\,dx$

RESOLUCIÓN

a) Efectuamos la división y expresamos el resultado de la forma $\frac{D}{d} = C + \frac{R}{d}$:

$$\int \frac{x^4 - 3x^2 + 2x - 2}{x - 1}\,dx = \int \left(x^3 + x^2 - 2x + \frac{-2}{x-1}\right) dx = \frac{x^4}{4} + \frac{x^3}{3} - x^2 - 2ln\,|x - 1| + k$$

b) $$\int \frac{\sqrt[3]{x} + \sqrt{2x^2}}{\sqrt[4]{x}}\,dx = \int \left(\frac{\sqrt[3]{x}}{\sqrt[4]{x}} + \frac{\sqrt{2x^2}}{\sqrt[4]{x}}\right) dx = \int \left(x^{\frac{1}{3}-\frac{1}{4}} + \sqrt{2}\,x^{1-\frac{1}{4}}\right) dx =$$

$$= \int \left(x^{\frac{1}{12}} + \sqrt{2}\,x^{\frac{3}{4}}\right) dx = \frac{x^{13/12}}{13/12} + \sqrt{2}\,\frac{x^{7/4}}{7/4} + k = \frac{12\,\sqrt[12]{x^{13}}}{13} + \frac{4\sqrt{2}\cdot\sqrt[4]{x^7}}{7} + k$$

c) $$\int \frac{2sen\,x + 3e^x}{5}\,dx = \frac{-2cos\,x + 3e^x}{5} + k$$

Calcula las siguientes integrales:

1 a) $\int (x^4 - 3x^2 + 2x - 1)\,dx$ b) $\int (x^3 - 2x)\,dx$

2 a) $\int \left(\frac{3}{4}x^5 - \frac{2}{3}x^2 + \frac{1}{7}\right) dx$ b) $\int x^3\,(x + 5)\,dx = \int (x^4 + 5x^3)\,dx$

3 a) $\int (x - 2)(x^2 + 4x)\,dx$ b) $\int (x^2 - 3)^2\,dx = \int (x^4 - 6x^2 + 9)\,dx$

4 a) $\int (2x^2 - 3)^2\,dx$ b) $\int \frac{\sqrt[5]{x} - \sqrt[3]{x^2}}{\sqrt{x}}\,dx$

5 a) $\int \frac{x^2 - \sqrt{x}}{2x}\,dx$ b) $\int \left(\frac{1}{2}e^x - \frac{3}{4}cos\,x\right) dx$

6 a) $\int \frac{3}{cos^2 x}\,dx$ b) $\int \frac{3sen\,x + 2^x}{4}\,dx$

7 a) $\int \frac{3}{1 + x^2}\,dx$ b) $\int \frac{e^x + tg\,x}{3}\,dx$

8 a) $\int \left(\frac{3}{x} + \frac{x^3}{3} + \frac{2}{x^4}\right) dx$ b) $\int \left(3^x + \frac{1}{x^3} - \frac{2}{x}\right) dx$

EJERCICIO RESUELTO

Calcula estas integrales:

a) $\int x\sqrt{1+x^2}\,dx$ ***b)*** $\int \frac{2\,dx}{x\ln x}$ ***c)*** $\int \frac{5x+3}{x^2+1}\,dx$

RESOLUCIÓN

a) $\int x\sqrt{1+x^2}\;dx = \frac{1}{2}\int 2x\,(1+x^2)^{1/2}\,dx = \frac{1}{2}\,\frac{(1+x^2)^{3/2}}{3/2} + k = \frac{\sqrt{(1+x^2)^3}}{3} + k$

b) $\int \frac{2\,dx}{x\ln x} = 2\int \frac{1/x}{\ln x}\,dx = 2\ln|\ln x| + k$

c) $\int \frac{5x+3}{x^2+1}\,dx = \int\left(\frac{5x}{x^2+1} + \frac{3}{x^2+1}\right)dx = \frac{5}{2}\int\frac{2x}{x^2+1}\,dx + 3\int\frac{1}{1+x^2}\,dx =$

$= \frac{5}{2}\ln|x^2+1|\;3arc\,tg\,x + k$

Halla las siguientes integrales:

9 a) $\int 2x\,(x^2+1)^8\,dx$ b) $\int x\sqrt{x^2-1}\;dx$

10 a) $\int x\sqrt{1-3x^2}\;dx$ b) $\int \cos x\cdot(sen\,x)^5\,dx$

11 a) $\int sen\,x\,\cos^4 x\,dx$ b) $\int \frac{2x}{x^2+5}\,dx$

12 a) $\int \frac{x}{3x^2-2}\,dx$ b) $\int \frac{12x^2-4x}{4x^3-2x^2+1}\,dx$

13 a) $\int \frac{3x^2-x}{4x^3-2x^2+1}\,dx$ b) $\int \frac{\cos x}{sen^6 x}\,dx$

14 a) $\int \frac{\cos x}{sen\,x}\,dx$ b) $\int e^{6x+5}\,dx$

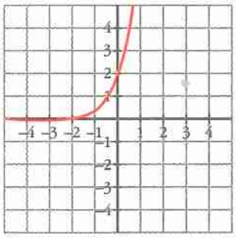
15 a) $\int 2x\,e^{x^2-3}\,dx$ b) $\int (x-1)\,e^{3x^2-6x}\,dx$

16 a) $\int \frac{e^{tg\,x}}{\cos^2 x}\,dx$ b) $\int (6x^3-1)\cos(3x^4-2x)\,dx$

17 a) $\int (2^{5x} - tg\,x)\,dx$ b) $\int 3\cos x\,e^{sen\,x}\,dx$

18 a) $\int \frac{e^{2x}}{e^{2x}+3}\,dx$ b) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

c)



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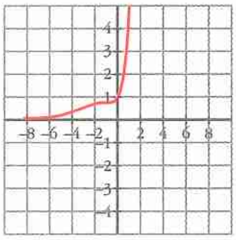

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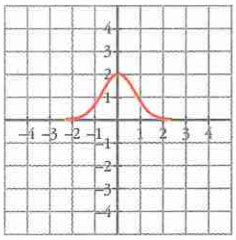

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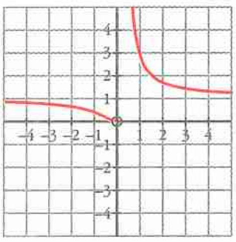

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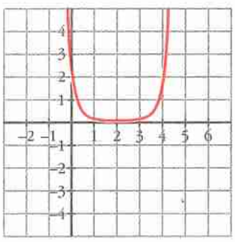

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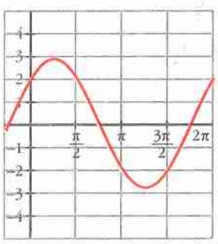

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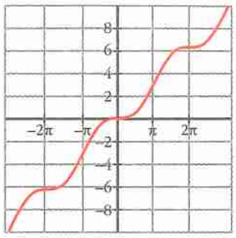

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1 a) $\frac{x^5}{5} - x^3 + x^2 - x + k$

b) $\frac{x^4}{4} - x^2 + k$

2 a) $\frac{x^6}{8} - \frac{2x^3}{9} + \frac{x}{7} + k$

b) $\frac{x^5}{5} + \frac{5x^4}{4} + k$

3 a) $\int (x^3 + 2x^2 - 8x)\, dx = \frac{x^4}{4} + \frac{2x^3}{3} - 4x^2 + k$

b) $\frac{x^5}{5} - 2x^3 + 9x + k$

4 a) $\int (4x^4 - 12x^2 + 9)\, dx = \frac{4x^5}{5} - 4x^3 + 9x + k$

b) $\int \left(x^{-3/10} - x^{1/6}\right) dx = \frac{10\sqrt[10]{x^7} - 6\sqrt[6]{x^7}}{7} + k$

5 a) $\frac{x^2}{4} - \sqrt{x} + k$

b) $\frac{1}{2}e^x - \frac{3}{4}\, sen\, x + k$

6 a) $3\, tg\, x + k$

b) $\frac{-3\cos x}{4} + \frac{2^x}{4\ln 2} + k$

7 a) $3\, arc\, tg\, x + k$

b) $\frac{e^x - \ln|\cos x|}{3} + k$

8 a) $3\ln|x| + \frac{x^4}{12} - \frac{2}{3x^3} + k$

b) $\frac{3^x}{\ln 3} - \frac{1}{2x^2} - 2\ln|x| + k$

BEREALAKO INTEGRALAK

9 a) $\int 2x\cdot(x^2+1)^8\,dx = \frac{1}{9}\int 9\cdot 2x(x^2+1)^8\,dx = \boxed{\frac{1}{9}\cdot(x^2+1)^9+k}$

$(x^2+1)^9 \longrightarrow 9(x^2+1)\cdot 2x$

b) $\int x\cdot\sqrt{x^2-1}\,dx = \frac{1}{3}\int 3x\cdot\sqrt{x^2-1}\,dx = \frac{1}{3}(x^2-1)^{3/2}+k = \boxed{\frac{1}{3}\sqrt{(x^2-1)^3}+k}$

$(x^2-1)^{1/2+1} \longrightarrow \frac{3}{\cancel{2}}(x^2-1)^{1/2}\cdot\cancel{2}x$

10 a) $\int x\cdot\sqrt{1-3x^2}\,dx = \frac{1}{(-9)}\int(-9)x\sqrt{1-3x^2}\,dx = -\frac{1}{9}(1-3x^2)^{3/2}+k = \boxed{-\frac{1}{9}\sqrt{(1-3x^2)^3}+k}$

$(1-3x^2)^{3/2} \longrightarrow \frac{3}{2}\cdot(1-3x^2)^{1/2}(-6x)$

$-9x$

b) $\int \cos x\cdot(\sin x)^5\,dx$

$(\sin x)^6 \longrightarrow 6(\sin x)^5\cdot\cos x$

$= \frac{1}{6}\int 6\cos x\cdot(\sin x)^5\cdot dx = \boxed{\frac{1}{6}\cdot\sin^6 x+k}$

11 a) $\int \sin x\cdot\cos^4 x\,dx.$

$\cos^5 x \longrightarrow -5\cdot\cos^4 x\cdot\sin x$

$= \frac{1}{5}\int -5\sin x\cdot\cos^4 x\cdot dx = \boxed{-\frac{1}{5}\cos^5 x+k}$

b) $\int \frac{2x}{x^2+5}\,dx = \boxed{\ln|x^2+5|+k}$

12) a) $\int \frac{x}{3x^2-2}\,dx =$ $\qquad \ln(3x^2-2) \to \frac{6x}{3x^2-2}$.

$= \frac{1}{6}\int \frac{6x}{3x^2-2}\,dx = \boxed{\frac{1}{6}\ln|3x^2-2|+K.}$

b) $\int \frac{12x^2-4x}{4x^3-2x^2+1}\,dx = \boxed{\ln|4x^3-2x^2+1|+K}$

13) a) $\frac{1}{4}\int \frac{4\cdot 3x^2-x}{4x^3-2x^2+1}\,dx = \boxed{\ln|4x^3-2x^2+1|+K}$ $\qquad \ln(4x^3-2x^2+1) \to \frac{12x^2-4x}{4x^3-2x^2+1}$

b) $\int \frac{\cos x}{\sin^6 x}\,dx = \int \frac{\cancel{\cos x}}{t^6}\cdot\frac{dt}{\cancel{\cos x}}$ $\qquad$!! $\cancel{\ln(\sin^6 x) \to \frac{6\sin^5 x\cos x}{\sin^6 x}}$

!! ORDEZKAPEN METODOA

$\sin x = t$

$\cos x\,dx = dt$

$dx = dt/\cos x.$

$= \int \frac{dt}{t^6} = \int t^{-6}\,dt = \frac{t^{-5}}{-5}+K$

$= \boxed{-\frac{1}{5\sin^5 x}+K.}$

14.) a) $\int \frac{\cos x}{\sin x}\,dx = \boxed{\ln|\sin x|+K.}$

b) $\int e^{6x+5}\,dx = \frac{1}{6}\int 6 e^{6x+5}\,dx =$ $\qquad e^{6x+5} \longrightarrow 6\cdot e^{6x+5}$

$= \boxed{\frac{1}{6}e^{6x+5}+K}$

15.) a) $\int 2x\,e^{x^2-3}\,dx = \boxed{e^{x^2-3}+K}$ $\qquad e^{x^2-3} \longrightarrow 2x\,e^{x^2-3}$

b) $\int (x-1)\cdot e^{3x^2-6x}\,dx =$ $\qquad e^{3x^2-6x} \longrightarrow (6x-6)e^{3x^2-6x}$

$6\cdot(x-1)\,e^{3x^2-6x}$

$= \frac{1}{6}\int 6(x-1)\cdot e^{3x^2-6x}\,dx = \boxed{\frac{1}{6}e^{3x^2-6x}+K}$

16) a) $\displaystyle\int \frac{e^{tg\,x}}{\cos^2 x}\,dx = \boxed{e^{tg\,x}+k}$

$e^{tg\,x} \longrightarrow \dfrac{e^{tg\,x}}{\cos^2 x}$

b) $\displaystyle\int (6x^3-1)\cos(3x^4-2x)\,dx$

$\sin(3x^4-2x) \longrightarrow \cos(3x^4-2x)(12x^3-2)$

$2(6x^3-1)$

$\displaystyle = \frac{1}{2}\int 2(6x^3-1)\cdot\cos(3x^4-2x)\,dx = \boxed{\frac{1}{2}\sin(3x^4-2x)+k}$

17) a) $\displaystyle\int (2^{\sqrt{x}} - tg\,x)\,dx$

$2^{\sqrt{x}} \longrightarrow 2^{\sqrt{x}}\ln 5.$

$\ln\cos x \longrightarrow \dfrac{-\sin x}{\cos x}$

$\displaystyle = \frac{1}{\ln 5}\int \ln 5\, 2^{\sqrt{x}}\,dx - (-)\int (-)\frac{\sin x}{\cos x}\,dx = \boxed{\frac{1}{\ln 5} + \ln(\cos x)+k.}$

b) $\displaystyle\int 3\cdot\cos x\cdot e^{\sin x}\,dx$

$e^{\sin x} \longrightarrow e^{\sin x}\cdot\cos x.$

$\displaystyle = 3\int \cos x\; e^{\sin x}\,dx = \boxed{3\cdot e^{\sin x}+k.}$

18) a) $\displaystyle\int \frac{e^{2x}}{e^{2x}+3}\,dx =$

$\ln(e^{2x}+3) \Longrightarrow \dfrac{2\cdot e^{2x}}{e^{2x}+3}$

$\displaystyle = \frac{1}{2}\int \frac{2\,e^{2x}}{e^{2x}+3}\,dx = \boxed{\frac{1}{2}\ln(e^{2x}+3)+k.}$

b) $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx =$

$e^{\sqrt{x}} \longrightarrow \dfrac{e^{\sqrt{x}}}{2\sqrt{x}.}$

$\displaystyle = 2\int \frac{e^{\sqrt{x}}}{2\cdot\sqrt{x}} = \boxed{2\cdot e^{\sqrt{x}}+k.}$